Received: *December 22, 1999*, Accepted: *February 24, 2000*

String creation and monodromy from fractional D-branes on ALE spaces

Tadashi Takayanagi

Department of Physics, Faculty of Science, University of Tokyo
Tokyo 113-0033, Japan
E-mail: takayana@hep-th.phys.s.u-tokyo.ac.jp

Abstract: We investigate the anomalous creation of fundamental strings using the boundary state formalism of fractional D-branes on ALE spaces in the orbifold limit. The open string Witten index plays a crucial role in this calculation and so the result remains unchanged even if we blow up the orbifold geometrically, matching the anomaly inflow argument. Further we consider the quiver gauge theories on such fractional D3-branes and see that the string creation mechanism determines 1-loop logarithmic monodromy of these gauge theories. Also we comment on the relation of D(-1)-D3 amplitude to the 1-loop beta function.

Keywords: D-branes, Conformal Field Models in String Theory, Brane Dynamics in Gauge Theories.

JHEP02(2000)040

Contents

1. Introduction

Since the discovery of D-branes [1], nonperturbative aspects of superstring theory have been intensively investigated and the tremendous progress has been made.

Usually, a D-brane is defined as a solitonic object to which open strings attach. But there is another equivalent description from the viewpoint of closed strings and it is called the boundary state [13]. The correspondence between these two descriptions which is called the Cardy's condition [14] gives a very crucial consistency condition if one would verify the existence of some complicated D-branes such as non-BPS D-branes discussed in [24] and D-branes in orbifold models or Gepner models discussed in [8, 9, 11, 10].

Another situation where the boundary state description of D-branes plays an important role is the investigation of string creation mechanism as already discussed in the case of flat space [17].

In this paper we discuss the string creation phenomenon in the case of the orbifold limit of ALE spaces and show that the Witten index of ALE space part of the open string CFT plays crucial role. So the result will not change if we blow up the orbifold. Further we derive the 1-loop monodromy of the quiver gauge theory on D-branes at an orbifold singularity.

This paper is organized as follows. In section 2, we review the description of fractional D-branes on the orbifold limit of ALE spaces and some mathematical facts. In section 3, we explain already known boundary state formalism of orbifold theory and give complete proof that this is indeed the case. In section 4, we calculate the open string Witten index by using boundary state formalism. In section 5, we investigate the string creation phenomenon of D-branes on ALE spaces from the viewpoint of both the CFT calculation and anomaly inflow. In section 6, we consider the quiver gauge theories on D-branes and show 1-loop monodromies are equivalent to the string creation phenomenon. In section 7, we summarize the conclusions and comment on future directions. In the appendix A, we give a explicit construction of boundary states of fractional D-branes at A_{N-1} singularity.

2. Fractional D-branes at orbifold singularities

Here we summarize the description of the fractional D-branes at the orbifold $\mathbb{C}^2/\mathbf{\Gamma}$ singularities [2, 3]. We concentrate on the orbifold limit of ALE spaces in this and the next section. But as we will see later, the main parts of the results of this paper remain correct if we blow up the orbifold.

The orbifold singularities $\mathbb{C}^2/\mathbf{\Gamma}$ are classified by the discrete group $\mathbf{\Gamma}$ falling into A, D, E series. Irreducible representations of $\mathbf{\Gamma}$ are finite dimensional (in particular one dimensional for A series) and we write them as $\{\rho_i\}$ $(1 \leq i \leq r_\Gamma)$,where we define r_Γ as the number of irreducible representations.

Explicit $\mathbf{\Gamma}$-actions on $\mathbb{C}^2$ whose coordinates are written as (z_1, z_2) are given in [2, 3] and for A_N type ($\mathbf{\Gamma} = \mathbb{Z}_{N+1}$) they are as follows

$$\begin{aligned} &\mathbf{\Gamma} = \{1, g, \ldots, g^N\} \qquad (g^{N+1} = 1) \\ &g : z_1 \to e^{\frac{2\pi i}{N+1}} z_1 \,, \quad z_2 \to e^{-\frac{2\pi i}{N+1}} z_2 \,. \end{aligned} \tag{2.1}$$

This two dimensional representation is fundamental and is called as the "natural representation" ρ_{nat}.

The geometric meaning of $\mathbb{C}^2/\mathbf{\Gamma}$-singularities is that some 2-cycles vanish at the origin and if we blow up those singularities we can define their intersection numbers $C_{\alpha\beta}$ for 2-cycle $[\alpha]$ and $[\beta]$. The following two mathematical facts are very useful.

- There is a one-to-one correspondence between homology 2-cycles and the irreducible representations :$[\alpha] \leftrightarrow \rho_\alpha$. Further in the decomposition $\rho_\alpha \otimes \rho_{nat} = \oplus_{\beta=1}^{r_\Gamma} a_{\alpha\beta}\rho_\beta$ we can identify $C_{\alpha\beta} = -2\delta_{\alpha\beta} + a_{\alpha\beta}$. It should also be mentioned that 2-cycles $\{[\alpha]\}$ $(1 \leq \alpha \leq r_\Gamma)$ are not all independent and follow the relation: $\sum_{\alpha=1}^{r_\Gamma} n_\alpha^{(0)}[\alpha] \sim 0$, where we set $n_\alpha^{(0)} = \dim(\rho_\alpha)$.

- $-2\delta_{\alpha\beta} + a_{\alpha\beta}(= C_{\alpha\beta})$ are equal to the Cartan matrices of the corresponding extended Lie algebras: $\hat{\mathbf{\Gamma}} = \{\hat{A}_N, \hat{D}_N, \hat{E}_6, \hat{E}_7, \hat{E}_8\}$. This highly nontrivial relation between the irreducible representations of $\mathbf{\Gamma}$ and extended Lie algebras is called as McKay correspondence [22].

Physical models at $\mathbb{C}^2/\mathbf{\Gamma}$-singularities are obtained from the conventional orbifold method and further we can consider D-branes in this theory. The orbifold CFT's are not really singular because there are twisted NSNS-B fields [4] on each of 2-cycles and wrapped strings and D-branes become massive. The explicit values of B-fields are given [5, 18] as

$$\int_{[\alpha]} B_{NS} = 2\pi \frac{n_\alpha^{(0)}}{|\Gamma|}, \qquad (n_\alpha^{(0)} = \dim(\rho_\alpha))\,. \tag{2.2}$$

A Dp-brane systems on $\mathbf{C^2}/\mathbf{\Gamma}$ space are classified by the representations of $\mathbf{\Gamma}$ action on their Chan-Paton factors Λ and we describe them as $\rho = \oplus_{\alpha=1}^{r_\Gamma} n_\alpha \rho_\alpha$. A D$p$-brane corresponding to ρ_α is called a (α-type) fractional Dp-brane and can be interpreted as a D(p+2)-brane wrapped on a vanishing 2-cycle $[\alpha]$ as discussed in [6]. We can always decompose a general Dp-brane system into fractional Dp-branes.

The spectrums of open strings attached to such a Dp-brane system are given as follows [2]. We define P_g-action on α-β string, which means the open string stretched between α-type and β-type fractional Dp-brane as

$$P_g : \Lambda \rightarrow \gamma^\alpha(g) \Lambda \gamma^\beta(g)^{-1} \cdot R(g)\,, \tag{2.3}$$

where we define $\gamma^\alpha(g)$ as the action $g \in \mathbf{\Gamma}$ on the Chan-Paton factor of ρ_α representation and $R(g)$ as the action $g \in \mathbf{\Gamma}$ on the oscillator part of the open strings.

If we concentrate on the massless modes, then we get 4D $N=2$ super Yang-Mills theory for $p=3$ and summarized as a quiver diagram [2]. Its gauge groups are $\prod_{\alpha=1}^{r_\Gamma} U(n_\alpha)$ and hyper multiplets belong to $\oplus_{\alpha,\beta} a_{\alpha,\beta}(\bar{\mathbf{n}}_\alpha, \mathbf{n}_\beta)$ representation. And the geometrical blowing up modes correspond to FI-terms in this Yang-Mills theory.

In particular, a "bulk" D-brane corresponds to the regular representation, which is defined as $\rho_{reg} = \oplus_\alpha n_\alpha^{(0)} \rho_\alpha$. The 4D $N=2$ super Yang-Mills theory on such branes have vanishing β-function implying superconformal invariance if we ignore each U(1) factor as discussed in [18].

Generally, using DBI-action and WZ-terms of D-branes we can identify [18] the complex gauge coupling of each $U(n_\alpha)$ part as

$$\tau_\alpha = \frac{\tau}{2\pi} \int_{[\alpha]} B_{NS} + \frac{1}{2\pi} \int_{[\alpha]} B_R\,, \qquad \tau = ie^{-\phi} + \chi\,, \tag{2.4}$$

where B_{NS},B_R means NSNS,RR 2-form field and ϕ,χ means dilaton,axion.

In the case of the regular representation, we get $\tau_\alpha = \frac{n_\alpha^{(0)}}{N}\tau = \text{const}$.

3. The boundary states of D-branes at orbifold singularities

The boundary states of fractional D-branes at general orbifold singularities have already been constructed in [8], but it seems that no complete proof have been given until now. So we give the proof here in the case of $\mathbb{C}^2/\mathbf{\Gamma}$ orbifold.[1]

First we define the character $\chi^\alpha(g)$ of g-action on the ρ_α representation as $\chi^\alpha(g) = \mathrm{Tr}\ \gamma^\alpha(g)$.

The boundary states of orbifold theories generally exist for each twisted sector. Let us define $|T_g\rangle$ $(g \in \mathbf{\Gamma})$ as

$$2\int_0^\infty \frac{dt}{2t}\,\mathrm{Tr}^{\mathrm{open}}_{\mathrm{NS-R}}\,\frac{g}{|\Gamma|}\frac{1+(-1)^F}{2}e^{-2\pi t H_o} = \langle T_g|\Delta|T_g\rangle\,, \tag{3.1}$$

where Tr means trace over zero modes and oscillators, and $\frac{1+(-1)^F}{2}$ and H_o correspond to the GSO-projection and the hamiltonian of the open string. And we defined the closed string propagator Δ as

$$\Delta = \pi\alpha' \int_0^\infty dl e^{-2\pi l H_c} \ \sim \frac{1}{k^2} + \cdots, \tag{3.2}$$

where H_c means the hamiltonian of the closed string.

Note that $|T_g\rangle$ is composed of NSNS-sector and RR-sector boundary state as $|T_g\rangle = |T_g\rangle_{\mathrm{NSNS}} + |T_g\rangle_{RR}$ and for each p ($p =$ even in IIA, $p =$ odd in IIB) we can construct corresponding $|T_g\rangle$. In appendix A we give the explicit forms of $|T_g\rangle$ for A_N-type orbifold.[2]

Every boundary state can be written as a linear combination of $\{|T_g\rangle\}$ $g \in \mathbf{\Gamma}$. Its coefficients are determined by the Cardy's condition [14] which requires that the $\mathbf{\Gamma}$-projections are well defined in open string sector.

So let us write down the Cardy's condition in the case of the open string stretched between fractional Dp-branes. Let us call the boundary state of a α-type fractional D-brane as $|Dp(\alpha)\rangle$. The correct $\mathbf{\Gamma}$-projection for $\alpha - \beta$ string is given as follows.

$$\begin{aligned}
&2\int_0^\infty \frac{dt}{2t}\tilde{\mathrm{Tr}}^{\mathrm{open}}_{\mathrm{NS-R}}\left[\frac{1+(-1)^F}{2}\mathcal{P}\, e^{-2\pi t H_o}\right] = \langle Dp(\alpha)|\Delta|Dp(\beta)\rangle\,,\\
&\mathcal{P} = \sum_{g\in\Gamma}\frac{P_g}{|\Gamma|} = \sum_{g\in\Gamma}\frac{P_g^{\mathrm{CP}}\cdot g}{|\Gamma|}\,,\\
&P_g^{\mathrm{CP}}:\ \Lambda \to \gamma^\alpha(g)\Lambda\gamma^\beta(g)^{-1}\,,
\end{aligned} \tag{3.3}$$

where $\tilde{\mathrm{Tr}}$ means trace over Chan-Paton factor in addition to zero modes and oscillators.

[1] As is easily seen, generalizations of this proof to other orbifold cases are straightforward.
[2] The similar calculations were carried out previously in ref [7].

If we first take the trace over only Chan-Paton factor, then we get

$$\text{l.h.s.} = 2\int_0^\infty \frac{dt}{2t}\,\mathrm{Tr}^{\text{open}}_{\text{NS}-\text{R}}\left[\left(\frac{1+(-1)^F}{2}\right)\left(\sum_{g\in\Gamma}\frac{g\chi^{\alpha *}(g)\chi^{\beta}(g)}{|\Gamma|}\right)e^{-2\pi t H_o}\right]. \tag{3.4}$$

For example, in the case of A_N series we can simplify this as follows.

$$\text{l.h.s.} = 2\int_0^\infty \frac{dt}{2t}\,\mathrm{Tr}^{\text{open}}_{\text{NS}-\text{R}}\left[\frac{1+(-1)^F}{2}\sum_{i=0}^{N}\frac{(g\xi^{\beta-\alpha})^i}{N+1}e^{-2\pi t H_o}\right], \tag{3.5}$$

where we defined $\xi = \exp(\frac{2\pi i}{N+1})$ and we regarded g as the generator of $A_N = \mathbb{Z}_{N+1}$.

Now, we can write down $|Dp(\alpha)\rangle$ solving the equation (3.3) as

$$|Dp(\alpha)\rangle = \sum_{g\in\Gamma}\chi^{\alpha}(g)|T_g\rangle. \tag{3.6}$$

This boundary state [8] represents a fractional Dp-brane which belongs to ρ_α representation and can be interpreted as a D$(p+2)$-brane wrapped on vanishing $[\alpha]$-cycle.

The tension and RR-charges of such a Dp-brane are easily obtained from this boundary state. To see this, we consider a bulk D-brane which belongs to the regular representation. The corresponding boundary state is

$$\begin{aligned} |Dp(reg)\rangle &= \sum_{\alpha,g} n_\alpha^{(0)}\chi^{\alpha}(g)|T_g\rangle \\ &= \sum_{\alpha}\chi^{\alpha *}(1)\chi^{\alpha}(g)|T_g\rangle \\ &= |\Gamma||T_1\rangle. \end{aligned} \tag{3.7}$$

Here we have used the following character formula

$$\sum_{\alpha\in\text{irr. rep.}}\chi^{\alpha *}(\mathcal{C}_i)\chi^{\alpha}(\mathcal{C}_j) = \frac{|\Gamma|}{h_i}\delta_{ij}\,, \tag{3.8}$$

where h_i is the number of elements which belong to the conjugacy class $\mathcal{C}_i$ $(i=1\sim r_\Gamma)$.

Using the technique developed in [15], we can conclude that a fractional Dp-brane of α-type has $1/|\Gamma|$ tension of a bulk Dp-brane and its g-th twisted sector RR-charges are $\chi^{\alpha}(g)/|\Gamma|$.

4. Open string Witten index

Here we concentrate on the internal 4D $(\mathbb{C}^2/\mathbf{\Gamma})$ CFT part $|B_\alpha\rangle$ of the full boundary state $|Dp(\alpha)\rangle$ and consider the open string Witten index. This 4D part contains all informations about the geometry of the orbifold limit ALE space. If we blow up the orbifold, then we will get different physical amplitudes, but topological quantities such as Witten index will not change.

Following [11, 12], let us define the open string Witten index: $\mathrm{tr}_{\alpha\beta}(-1)^F$ as

$$\begin{aligned}\mathrm{tr}_{\alpha\beta}(-1)^F &= \mathrm{Tr}_R^{\alpha-\beta\mathrm{string}}[(-1)^F e^{-2\pi t H_o}] \\ &= \mathrm{Tr}_R^{\mathrm{open}}\left[(-1)^F \sum_{g\in\Gamma} \frac{g\chi^{\alpha*}(g)\chi^{\beta}(g)}{|\Gamma|}\, e^{-2\pi t H_o}\right].\end{aligned} \tag{4.1}$$

Using the Cardy's condition, we get

$$2\int_0^\infty \frac{dt}{2t}\,\mathrm{tr}_{\alpha\beta}\,(-1)^F = \langle B_\alpha, +|\Delta|B_\beta, -\rangle_{RR}\,, \tag{4.2}$$

where $+, -$ means the spin structure of the boundary state (see appendix A or [13]).

As usual, massive modes (non chiral primary states) have no effect on this Witten index. So we can explicitly calculate this as follows

$$\begin{aligned}\mathrm{tr}_{\alpha\beta}\ (-1)^F &= -2\sum_{g\in\Gamma}\frac{\chi^{\alpha*}(g)\chi^{\beta}(g)}{|\Gamma|} + \sum_{g\in\Gamma}\frac{\chi^{\alpha*}(g)\chi^{\beta}(g)\chi^{nat}(g)}{|\Gamma|} \\ &= -2\delta_{\alpha\beta} + a_{\alpha\beta} \\ &= C_{\alpha\beta}\,.\end{aligned} \tag{4.3}$$

Here we have used the decomposition

$$\rho_\alpha \otimes \rho_{nat} = \oplus_{\beta=1}^{r_\Gamma} a_{\alpha\beta}\rho_\beta\,, \tag{4.4}$$

and the following formulas

$$\begin{aligned}\sum_{g\in\Gamma}\chi^{\alpha*}(g)\chi^{\beta}(g) &= |\Gamma|\delta_{\alpha\beta}\,, \\ \chi^{\rho\otimes\rho'}(g) &= \chi^{\rho}(g)\cdot\chi^{\rho'}(g)\,.\end{aligned} \tag{4.5}$$

5. String creation from D-branes on ALE spaces

5.1 CFT calculations

Here we investigate D2-D6 system each wrapped on the 2-cycle of ALE spaces and show the anomalous creation of string in that system. In our orbifold description such a system is regarded as fractional D0-D4 system.

First let us calculate the amplitudes of $D0-D4$ and $\bar{D}0-D4$ string. $\bar{D}0$ means anti D0-brane and its boundary state has an extra minus sign in front of the RR-sector boundary state. We assume that a fractional $D0$ or $\bar{D}0$-brane is α-type and a fractional $D4$-brane is β-type.

JHEP02(2000)040

Then we get

$$
\begin{aligned}
&\langle D0(\alpha)|\Delta|D4(\beta)\rangle = \\
&= -2\int_0^\infty \frac{dt}{2t}\sum_{g\in\Gamma}\chi^{\alpha *}(g)\chi^\beta(g)\,\mathrm{Tr}^{\mathrm{open}}_{\mathrm{NS-R}}\left[\frac{1\pm(-1)^F}{2}\frac{g}{|\Gamma|}\;e^{-2\pi H_o t}\right] \\
&= -V_1\int_0^\infty \frac{dt}{2t}\sum_{g\in\Gamma}\chi^{\alpha *}(g)\chi^\beta(g)\times \\
&\qquad \times\int\left(\frac{dk}{2\pi}\right)e^{-\frac{y^2 t}{2\pi\alpha'}-2\pi\alpha' t k^2}\hat{\mathrm{Tr}}^{\mathrm{open}}_{\mathrm{NS-R}}\left[(1\pm(-1)^F)\frac{g}{|\Gamma|}e^{-2\hat{H}_o t}\right], \qquad (5.1)
\end{aligned}
$$

where we choose[3] $-,+$ sign for $D0$-brane,$\bar{D}0$-brane, respectively and superscript $\hat{*}$ means the elimination of bosonic zero modes. V_1 means the volume of D0-brane and y is the distance between D0 and D4.

From these amplitudes we can read off the potential energy V of D0-D4 system. Naively, one may expect that V should vanish because of supersymmetry in this system. To see unbroken supersymmetries, let us define by $(x^0, x^1, x^2, x^3, x^4)$ the world volume of D4-brane and by (x^6, x^7, x^8, x^9) the ALE space. Further we write Γ-matrices and a linear combination of super-charges as $\{\Gamma_\mu\}$ and $\epsilon_L Q_L + \epsilon_R Q_R$. Then the unbroken supersymmetries are given as

$$
\begin{aligned}
D4 &: \epsilon_L = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\epsilon_R\,, \\
D0, \bar{D}0 &: \epsilon_L = \pm\Gamma^0\epsilon_R\,, \\
ALE &: \epsilon_L = -\Gamma^6\Gamma^7\Gamma^8\Gamma^9\epsilon_L\,,\ \epsilon_R = -\Gamma^6\Gamma^7\Gamma^8\Gamma^9\epsilon_R\,. \qquad (5.2)
\end{aligned}
$$

So we get unbroken 1/8 supersymmetry (= 4 SUSY).

But explicit CFT calculation of (5.1) shows that such an expectation is partly wrong as follows

$$
\begin{aligned}
&V(\bar{D}0 - D4) - V(D0 - D4) = \\
&\quad= \frac{1}{V_1}\langle\bar{D}0(\alpha)|\Delta|D4(\beta)\rangle - \frac{1}{V_1}\langle D0(\alpha)|\Delta|D4(\beta)\rangle \\
&\quad= \int_0^\infty \frac{dt}{t}\sum_{g\in\Gamma}\chi^\alpha(g)\chi^\beta(g)\frac{1}{\sqrt{8\pi^2\alpha' t}}e^{-\frac{y^2 t}{2\pi\alpha'}}\;\hat{\mathrm{Tr}}_R(-1)^F\frac{g}{|\Gamma|}e^{-2\hat{H}_o t} \\
&\quad= \int_0^\infty \frac{dt}{t}\frac{1}{\sqrt{8\pi^2\alpha' t}}\;e^{-\frac{y^2 t}{2\pi\alpha'}}\;\mathrm{tr}_{\alpha\beta}(-1)^F \\
&\quad= -\tau_{F1}C_{\alpha\beta}\,y \qquad \left(\tau_{F1} = \frac{1}{2\pi\alpha'}\right), \qquad (5.3)
\end{aligned}
$$

In this calculation we have used the fact $\hat{\mathrm{Tr}}_{NS}\,(-1)^F = 0$ (because of fermionic zero modes) and $\mathrm{tr}_{\alpha\beta}(-1)^F = C_{\alpha\beta}$ as shown in previous section.

[3]This is a matter of convention.

Further, we argue

$$V(D0 - D4) = 0\,. \tag{5.4}$$

Eq. (5.4) can be easily verified in the lowest order and will be correct due to the supersymmetry. So we get

$$V(\bar{D}0 - D4) = -\tau_{F1} C_{\alpha\beta}\, y\,, \tag{5.5}$$

and this implies that $\bar{D}0 - D4$ system is not supersymmetric.

At first sight, it does not seem natural that this result does not treat $D0$ and $\bar{D}0$ symmetrically. But once we fix the convention to determine which corresponds to the $D0$ or $\bar{D}0$ in eq. (5.1), this holds by explicit CFT calculations and the similar anomalous situation can also be seen in the familiar $\bar{D}0 - D8$ system discussed in [17], which implies that the string creation phenomenon is expected also in our case.

Indeed if we interpret $\bar{D}0 - D4$ system as the rotated D0-D4 system by π in the plane spanned by the direction of D0-brane and the transverse direction x^5 , then the energy difference (5.3) shows that $C_{\alpha\beta}$ fundamental strings are created when a D0-brane of α-type crosses a D4-brane of β-type.[4] This result will be the same if we T-dualize D0-D4 system into Dp-D$(4-p)$ system.

It is important to note that in this calculation the open string Witten index is crucial and if we blow up the orbifold into various ALE spaces or more globally K3 surfaces, the result will not change. So we get the following conclusion taking the T-duality equivalence into consideration.

When a Dp-brane wrapped on a 2-cycle $[\alpha]$ crosses a D$(8-p)$-brane wrapped on a 2-cycle $[\beta]$, $C_{\alpha\beta}$ fundamental strings are created, where $C_{\alpha\beta}$ is the intersection number of $[\alpha]$ and $[\beta]$.

5.2 Anomaly inflow argument

Generally, string creation can also be investigated by using anomaly inflow argument developed in [16]. Anomalous creation of strings occurs if there are chiral fermions in the D-brane system such as $D5 - D5$ ($D5 \cap D5 =$1-brane) in flat space, which is equivalent by T-duality to $D0 - D8$ system.

Let us apply this method to our case: D2-brane wrapped on $[\alpha]$ cycle and D6-brane wrapped on $[\beta]$ cycle. First we T-dualize this D2-D6 system into D3-D7 system, then chiral fermions in T-dualized D3-D7 system are generated from the intersection of D-branes and the number of chiral fermions is $C_{\alpha\beta}$ including the sign of the orientation. So we get the following anomaly equation.

$$\Delta N = C_{\alpha\beta} R \int dt \left(\frac{dA^5_{(D3)}}{dt} - \frac{dA^5_{(D7)}}{dt} \right), \qquad (t = x^0)\,, \tag{5.6}$$

[4]If $C_{\alpha\beta} < 0$, then it means that a string disappears if there are already some strings before the D0-brane crosses the D4-brane and that a string with the opposite orientation is created if not.

where we have compactified x^5 direction (radius=R) and A_{D3}, A_{D5} means the gauge field on the D3,D7-brane.

Next we take T-dual in the x^5 direction again and we get

$$\Delta N = \frac{C_{\alpha\beta}}{2\pi R'} \int dt \, \frac{d(\Delta X_5)}{dt} , \qquad (t = x^0) , \tag{5.7}$$

where the radius of x^5 direction is $R' = \alpha'/R$ and ΔX_5 means the distance between D2 and D6.

This result is the same as that obtained in previous subsection by explicit CFT calculation. It will be important to note that the resolution by B-field is equivalent to the geometric resolution in the string creation phenomenon as the comparison between these two methods shows.

6. 1-loop monodromy of quiver gauge theories

6.1 String creation and monodromy

Here we consider fractional coincident D3-branes on the orbifold limit of ALE spaces in TypeIIB string theory. We call the number of each type of fractional D3-branes as $\{n_\alpha\}$. The quiver gauge theory on that system becomes $N = 2$ super Yang-Mills theory and so we consider 1-loop β-functions for each gauge group SU(n_α).[5] We define the complex gauge couplings as

$$\tau_\alpha^{YM} = i\frac{4\pi}{g_\alpha^2} + \frac{\theta_\alpha}{2\pi} . \tag{6.1}$$

We assume D3-brane world volume and 4D orbifold space extends in the direction of (x^0, x^1, x^2, x^3) and (x^6, x^7, x^8, x^9). The remaining (x^4, x^5)-plane will play the role of 'u-plane' or moduli space of the Coulomb branch.

Now let us apply the string creation mechanism discussed in the previous section to D1-D3 system, where a fractional D1-brane stretches in the direction of (x^0, x^4). If a D1-brane of α-type crosses the D3-brane system, then $\sum_{\beta=1}^{r_\Gamma} C_{\alpha\beta} n_\beta$ fundamental strings are created.

This means a monodromy about the scalar fields τ_α eq. (2.4) for each α around $(x^4, x^5) = (0, 0)$, where the D3-branes are coincident. Let us call a bound state of p fundamental strings and q_α fractional (α-type) D1-branes as (p, q_α) string. Then the string creation phenomenon in our case means that if the D1-brane crosses the D3-brane, a three string junction is generated and (p, q_α) string changes into $(p - q_\alpha(\sum_{\beta=1}^{r_\Gamma} C_{\alpha\beta} n_\beta),\ q_\alpha)$ at that point, creating $q_\alpha(\sum_{\beta=1}^{r_\Gamma} C_{\alpha\beta} n_\beta)$ fundamental strings. The shift of the value q_α is equally described as the shift of τ_α taking into consider-

[5] Here we ignored each U(1) factors for the reason discussed below.

ation the fact that the tension of a (p, q_α) string is proportional to $|q_\alpha \tau_\alpha + p|$. So we get the following monodromy.

$$\begin{aligned}\tau_\alpha &\to \tau_\alpha + \sum_{\beta=1}^{r_\Gamma} C_{\alpha\beta} n_\beta \\ \tau_\alpha &\sim \tau_\alpha^{(0)} + \frac{1}{2\pi i} \sum_{\beta=1}^{r_\Gamma} C_{\alpha\beta} n_\beta \log(z) \qquad (z = x^4 + ix^5)\,. \end{aligned} \tag{6.2}$$

Surprisingly, this logarithmic behavior is the same as that of the running gauge coupling τ_α^{YM} in the perturbation theory

$$\frac{\partial \tau_\alpha^{YM}(E)}{\partial(\log E)} = \frac{1}{2\pi i} \sum_{\beta=1}^{r_\Gamma} C_{\alpha\beta} n_\beta\,, \tag{6.3}$$

if we ignore each U(1) factor of the quiver gauge theory and identify z as the energy scale for SU(n_α) gauge theory.

This correspondence is understood by the following two reasons.

The first one is to apply the (slightly perturbed) AdS/CFT correspondence as discussed in [19, 18]. Remember that the gauge couplings are identified as the type-IIB background given in (2.4). In this interpretation we should naturally assume that the disturbance from the regular rep. is small and the large-N limit is taken. But the observed correspondence $\tau_\alpha \sim \tau_\alpha^{YM}$ holds without such an assumption.

The second one is to use the same argument discussed in [20]. In fact our brane system at a A_N-type orbifold singularity is equivalent to the NS5-D4 system (MQCD) by T-duality. $\mathrm{Im}\,\tau_\alpha$ corresponds to the distance between the α-th two NS5-branes. $\mathrm{Re}\,\tau_\alpha$ corresponds to the coordinate of S^1 which appears when the system is lifted to M-theory. The identification of z as the energy scale is also the same. The fractional D3-brane at the orbifold are sources for the scalar fields τ_α and if we take the T-duality, then the presence of such sources leads to the bending of NS5-branes. This interpretation holds without any restrictions about the gauge group and the decoupling of U(1) factors is also observed in [20].

6.2 1-loop β-function from D(-1)-D3 amplitude

Another way to see these 1-loop monodromies is to calculate the interaction between a fractional D(-1)-brane and the D3-brane system. This amplitude corresponds to the leading correction to the action of a D(-1)-brane in the presence of D3-branes as follows.

$$S_{\mathrm{inst}} = 2\pi i \tau_\alpha = 2\pi i \tau_\alpha'^{(0)} - \langle D(-1)|\Delta|D3\rangle + \cdots \tag{6.4}$$

Explicit calculation shows[6]

$$\begin{aligned}\langle D(-1)(\alpha)|\Delta|D3(\beta)\rangle &= \int_0^\infty \frac{dt}{2t}\, e^{-\frac{|z|^2 t}{2\pi\alpha'}} \left[\mathrm{tr}_{\alpha\beta}\,(-1)^F - \mathrm{tr}_{\alpha\beta}\,(-1)^F\right] = 0\\ \langle \bar{D}(-1)(\alpha)|\Delta|D3(\beta)\rangle &= \int_0^\infty \frac{dt}{2t}\, e^{-\frac{|z|^2 t}{2\pi\alpha'}} \left[\mathrm{tr}_{\alpha\beta}\,(-1)^F + \mathrm{tr}_{\alpha\beta}\,(-1)^F\right]\\ &\sim \text{const.} - 2C_{\alpha\beta}\log|z|\,. \end{aligned} \tag{6.5}$$

This seems to be inconsistent with eq. (6.2) and to be a non-sypersymmetric (not holomorphic) background. So this implies something is wrong with this boundary state calculation and the 2D harmonic functions which appear in (6.5) should be modified so as to reproduce eq. (6.2) as follows

$$\begin{aligned}\text{For NSNS-sector:}\quad &\int_0^\infty \frac{dt}{2t} e^{-\frac{|z|^2 t}{2\pi\alpha'}} \;=\; \text{const.} - \log|z| \qquad \text{(unchanged)}\\ \text{For RR-sector:}\quad &\int_0^\infty \frac{dt}{2t} e^{-\frac{|z|^2 t}{2\pi\alpha'}} \rightarrow \text{const.} + i\arg(z) \qquad \text{(changed)}\,. \end{aligned} \tag{6.6}$$

This subtlety will be peculiar to the calculation which involves 2D harmonic functions, where there are two different 2D harmonic functions: $\sim \log|z|$ and $\sim \arg(z)$.

Note that this derivation of the background around D3-branes can be seen as the generalization of the method developed in [15]. To see this, take the low energy limit, and we can regard $|D(-1)(\alpha)\rangle$ as the vertex operator of twisted NSNS and RR scalars. But it is important that in our case the result is exact if we ignore the nonperturbative effects, indicating the well known fact that 4D $N = 2$ gauge theories receive no higher loop corrections.

6.3 Comments on some other aspects of quiver gauge theories

First we consider instantons in these gauge theories. A D-instanton of α-type can be regarded as an instanton of $U(n_\alpha)$ gauge theory. Let us define the instanton numbers as

$$\frac{1}{32\pi^2}\int F_\alpha \tilde{F}_\alpha = k_\alpha\,. \tag{6.7}$$

The bosonic moduli of this instanton corresponds to the bosonic zero modes in NS-sector D(-1)-D3 open string. So we get

$$\dim \mathcal{M}_{\text{instanton}} = 4\sum_{\alpha=1}^{r_\Gamma} k_\alpha n_\alpha \tag{6.8}$$

as expected. Unfortunately, as for the fermionic zero modes we get into a puzzle because the number of zero modes in $D(-1)-D3$ is different from that in $\bar{D}(-1)-D3$. So we leave this as an open problem.

[6] D(-1)-D3 system is formally (time like) T-dual to D0-D4 system discussed before and the calculation is almost the same.

Next we consider chiral transformations. We define α-type chiral transformation which is analogous to R-symmetry as

$$\begin{aligned} \Phi_\alpha \to e^{2i\phi}\Phi_\alpha\,, \qquad W_\alpha \to e^{i\phi}W_\alpha\,, \qquad \theta \to e^{-i\phi}\theta \\ H_{\alpha\beta} \to H_{\alpha\beta}\,, \qquad \tilde{H}_{\alpha\beta} \to \tilde{H}_{\alpha\beta}\,, \end{aligned} \tag{6.9}$$

where (W_α, Φ_α) and $(H_{\alpha\beta}, \tilde{H}_{\alpha\beta})$ are chiral super field descriptions of vector multiplet ($U(n_\alpha)$ gauge) and hyper multiplet ($(\bar{\mathbf{n}}_\alpha, \mathbf{n}_\beta)$ rep.) and θ denotes the superspace fermionic coordinate. Here we restrict the value of β such that $C_{\alpha\beta} \neq 0$ and so this chiral transformation defined for each α is different from the conventional U(1) R-symmetry.

For $\phi = \pi$, this transformation becomes a classical symmetry of the quiver theory, but incorporating quantum effects this transformation changes the whole action as $S \to S + \Delta S$. Conventional anomaly argument shows

$$\partial_\mu J^\mu_{R\alpha} = -\frac{2}{32\pi^2}\left(\sum_\beta C_{\alpha\beta}n_\beta\right) F\tilde{F} \quad \to \quad \Delta S = \pi \left(\sum_\beta C_{\alpha\beta}n_\beta\right) \frac{1}{16\pi^2}\int F\tilde{F}\,. \tag{6.10}$$

So we get the following monodromy for each α

$$\Phi_\alpha \to e^{2\pi i}\Phi_\alpha\,, \qquad \theta_\alpha \;\to\; \theta_\alpha + 2\pi\sum_\beta C_{\alpha\beta}n_\beta. \tag{6.11}$$

This result matches (6.2) indicating the identification: $\Phi_\alpha \sim z_\alpha$.

We conclude this section by mentioning the nonperturbative correction to this quiver theory from our view point. Exact solutions to these theories are already given in [20] for T-dualized model of A_N case and so we only comment on what will happen to the D3-brane system nonperturbatively.

So far we assumed that the D3-branes are coincident at the origin, but the effect of instanton corrections may separate them and generate characteristic mass scales.

Let us consider the case $\sum_\beta C_{\alpha\beta}n_\beta < 0$ which includes pure $N = 2$ SU(n) Yang-Mills theory. In this case, (6.2) indicates $\tau(z_\alpha) \to -i\infty$ ($z_\alpha \to 0$) and the interpretation (6.1) does not work. This means that in this case D3-branes cannot be coincident and should 'fragment' by the instanton effects until the condition $\sum_\beta C_{\alpha\beta}n_\beta \geq 0$ is satisfied. Quite recently some dynamics of these situations are considered from the viewpoint of AdS/CFT correspondence [21].

7. Conclusions

In this paper we have investigated the boundary state descriptions of fractional D-branes at orbifold singularities and calculated some amplitudes of the open sting stretching between such D-branes which have the topological nature and are proportional to the open string Witten index.

JHEP02(2000)040

Such amplitudes show the occurrence of anomalous string creation when a fractional Dp-brane crosses a fractional D$(4-p)$-brane in the orbifold($\mathbf{C^2/\Gamma}$) type-II theory. This topological nature will not change if we blow up the orbifold and the same phenomenon will occur in type-II string theory compactified on K3 surfaces.

We have also mentioned the 1-loop monodromies in the quiver gauge theories on fractional D3-branes. Such monodromies are equivalent to the string creation phenomenon. But the interpretation of the fermionic zero modes of D(-1)-D3 string has been left as an open problem.

Our treatment covers all types of D3-brane systems at an orbifold singularity, but if we take the large-N limit, two important possibility can be considered. One possibility is that there are infinite many bulk D-branes which belong to the regular representation ($\sum_\beta C_{\alpha\beta}n_\beta = 0$) and 4D $N = 2$ superconformal field theories are realized on those systems as discussed in [18]. Another possibility is that there are many fractional D-branes which belong to one particular type ($\sum_\beta C_{\alpha\beta}n_\beta \to \infty$). This case will provide us very interesting 'exotic black holes'(or repulson) and AdS/CFT correspondence setups as discussed in [21]. The dynamical aspects of these situations should be studied systematically in the future.

It will be interesting to generalize these results to more general compactifications such as Calabi-Yau spaces. D-branes on particular Calabi-Yau spaces have already been discussed using the boundary state description in [8] for $\mathbb{C}^3/\mathbf{\Gamma}$ and [11, 10] for quintic surface. The string creation phenomenon in these examples will again closely be related to the open string Witten index and the intersection numbers and will be able to be discussed in the same way as our case. So such investigations will require the calculation of amplitudes at only particular points in the moduli spaces of such Calabi-Yau spaces. It will also be interesting to study those string creation mechanism from the viewpoint of gauge theories on the D-branes.

Acknowledgments

I am very grateful to T. Eguchi for valuable advises, helpful discussions and careful reading of the manuscript. I also thank S. Terashima, M. Naka and M. Nozaki for helpful discussions and remarks.

A. Explicit boundary states of A_{N-1} type orbifolds

Here we construct fractional D0-brane boundary states $|T_g\rangle$ of A_{N-1} type orbifolds explicitly by using the Cardy's condition. We represent an element of $\mathbf{\Gamma} = \mathbf{Z_N}$ as k $(0 \leq k \leq N-1)$.

CFT conventions. Let $(z, \bar{z})$ be world sheet coordinates and $(X^0 \sim X^5, Z_1, Z_2)$ $(\psi^0 \sim \psi^5, \eta, \xi)$ be the fields corresponding to spacetime coordinates and their super partners. As in [25], we use light cone formalism and we regard X^0, X^1 as the light cone direction. We set $a = 2, 3, 4, 5$ below.

Then the string fields in the k-th twisted sector can be expanded as

$$
\begin{aligned}
\partial X^a(z) &= -i\sqrt{\frac{\alpha'}{2}}\sum_{m\in\mathbb{Z}}\alpha^a_m z^{-m-1} & \bar\partial X^a(\bar z) &= -i\sqrt{\frac{\alpha'}{2}}\sum_{m\in\mathbb{Z}}\tilde\alpha^a_m \bar z^{-m-1}\\
\psi^a(z) &= \sum_r \psi^a_r z^{-r-\frac{1}{2}} & \tilde\psi^a(\bar z) &= \sum_r \tilde\psi^a_r \bar z^{-r-\frac{1}{2}}\\
\partial Z_1(z) &= -i\sqrt{\alpha'}\sum_{m\in\mathbb{Z}}\alpha_{m-\frac{k}{N}} z^{-m-1-\frac{k}{N}} & \bar\partial Z_1(\bar z) &= -i\sqrt{\alpha'}\sum_{m\in\mathbb{Z}}\tilde\alpha_{m+\frac{k}{N}} \bar z^{-m-1+\frac{k}{N}}\\
\partial \bar Z_1(z) &= -i\sqrt{\alpha'}\sum_{m\in\mathbb{Z}}\bar\alpha_{m+\frac{k}{N}} z^{-m-1+\frac{k}{N}} & \bar\partial \bar Z_1(\bar z) &= -i\sqrt{\alpha'}\sum_{m\in\mathbb{Z}}\tilde{\bar\alpha}_{m-\frac{k}{N}} \bar z^{-m-1-\frac{k}{N}}\\
\partial Z_2(z) &= -i\sqrt{\alpha'}\sum_{m\in\mathbb{Z}}\beta_{m+\frac{k}{N}} z^{-m-1+\frac{k}{N}} & \bar\partial Z_2(\bar z) &= -i\sqrt{\alpha'}\sum_{m\in\mathbb{Z}}\tilde\beta_{m-\frac{k}{N}} \bar z^{-m-1-\frac{k}{N}}\\
\partial \bar Z_2(z) &= -i\sqrt{\alpha'}\sum_{m\in\mathbb{Z}}\bar\beta_{m-\frac{k}{N}} z^{-m-1-\frac{k}{N}} & \bar\partial \bar Z_2(\bar z) &= -i\sqrt{\alpha'}\sum_{m\in\mathbb{Z}}\tilde{\bar\beta}_{m+\frac{k}{N}} \bar z^{-m-1+\frac{k}{N}}\\
\eta(z) &= \sqrt{2}\sum_r \eta_{r-\frac{k}{N}} z^{-r-\frac{1}{2}+\frac{k}{N}} & \eta(\bar z) &= \sqrt{2}\sum_r \tilde\eta_{r+\frac{k}{N}} z^{-r-\frac{1}{2}-\frac{k}{N}}\\
\xi(z) &= \sqrt{2}\sum_r \eta_{r+\frac{k}{N}} z^{-r-\frac{1}{2}-\frac{k}{N}} & \xi(\bar z) &= \sqrt{2}\sum_r \tilde\xi_{r-\frac{k}{N}} z^{-r-\frac{1}{2}+\frac{k}{N}},
\end{aligned}
\tag{A.1}
$$

where the modings: r of fermions are all half integer in NS-sector and integer in R-sector. And non trivial (anti)commutation relations are as follows (we only show left-moving parts only)

$$
\begin{aligned}
[\bar\alpha_{m+k/N}, \alpha_{n-k/N}] &= \left(m+\frac{k}{N}\right)\delta_{m,-n} & [\bar\beta_{m-k/N}, \beta_{n+k/N}] &= \left(m-\frac{k}{N}\right)\delta_{m,-n}\\
\{\eta_{r-k/N}, \bar\eta_{r+k/N}\} &= \delta_{r+s} & \{\xi_{r+k/N}, \bar\xi_{r-k/N}\} &= \delta_{r+s}\,.
\end{aligned}
\tag{A.2}
$$

So k-th twisted sector vacuum is defined as

$$
\begin{aligned}
\alpha_{m-k/N}|0\rangle_k &= 0\ (m>0) & \tilde\alpha_{m+k/N}|0\rangle_k &= 0\ (m\geq 0)\\
\bar\alpha_{m+k/N}|0\rangle_k &= 0\ (m\geq 0) & \tilde\alpha_{m-k/N}|0\rangle_k &= 0\ (m>0)\\
\beta_{m+k/N}|0\rangle_k &= 0\ (m\geq 0) & \tilde{\bar\alpha}_{m-k/N}|0\rangle_k &= 0\ (m>0)\\
\bar\beta_{m-k/N}|0\rangle_k &= 0\ (m>0) & \tilde\alpha_{m+k/N}|0\rangle_k &= 0\ (m\geq 0)\\
\eta_{r-k/N}|0\rangle_k &= 0\ (r>0) & \tilde\eta_{r+k/N}|0\rangle_k &= 0\ (r\geq 0)\\
\bar\eta_{r+k/N}|0\rangle_k &= 0\ (r\geq 0) & \tilde{\bar\eta}_{r-k/N}|0\rangle_k &= 0\ (r>0)\\
\xi_{r+k/N}|0\rangle_k &= 0\ (r\geq 0) & \tilde\xi_{r-k/N}|0\rangle_k &= 0\ (r>0)\\
\bar\xi_{r-k/N}|0\rangle_k &= 0\ (r>0) & \tilde{\bar\xi}_{r+k/N}|0\rangle_k &= 0\ (r\geq 0)\,.
\end{aligned}
\tag{A.3}
$$

Then the zero point energy is

$$
a_{NS} = -\frac{1}{2}+\frac{k}{N}\,, \qquad a_R = 0\,. \tag{A.4}
$$

and physical states satisfy $(L_0 + a_{\rm sector})|{\rm phys}\rangle = (\bar{L}_0 + a_{\rm sector})|{\rm phys}\rangle = 0$. Here we defined

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{a=2}^{5}\left(\sum_{n\geq 1}\alpha^a_{-n}\alpha^a_n + \sum_{r>0} r\psi^a_{-r}\psi^a_r\right) + \\ + \sum_{n=-\infty}^{\infty}\left(:\alpha_{n-\frac{k}{N}}\bar{\alpha}_{-n+\frac{k}{N}}: + :\beta_{n+\frac{k}{N}}\bar{\beta}_{-n-\frac{k}{N}}:\right) + \\ + \sum_{r=-\infty}^{\infty}\left\{:\left(r-\frac{k}{N}\right)\eta_{r-\frac{k}{N}}\bar{\eta}_{n+\frac{k}{N}}: + :\left(r+\frac{k}{N}\right)\xi_{r+\frac{k}{N}}\bar{\xi}_{r-\frac{k}{N}}:\right\}, \quad \text{(A.5)}$$

and $\tilde{L_0}$ similarly. For later use, we define closed string hamiltonian H_c as

$$H_c = L_0 + \tilde{L_0} + a^L_{\rm sector} + a^R_{\rm sector}\,. \quad \text{(A.6)}$$

and open string hamiltonian:H_o similarly.

Construction of boundary state. The D0-brane k-th twisted sector boundary state: $|T_k,\epsilon\rangle$ $(0 \leq k \leq N-1)$ in the theory of type-IIA on $R^{1,5}\times C^2/\mathbb{Z}_N$ is defined as follows.

$$\begin{aligned}
\left(\alpha^a_n - \tilde{\alpha}^a_{-n}\right)|T_k,\epsilon\rangle &= 0 & \left(\psi^a_r - i\epsilon\psi^a_{-r}\right)|T_k,\epsilon\rangle &= 0\\
\left(\alpha_{m-\frac{k}{N}} - \tilde{\alpha}_{-m+\frac{k}{N}}\right)|T_k,\epsilon\rangle &= 0 & \left(\bar{\alpha}_{m+\frac{k}{N}} - \tilde{\bar{\alpha}}_{-m-\frac{k}{N}}\right)|T_k,\epsilon\rangle &= 0\\
\left(\beta_{m+\frac{k}{N}} - \tilde{\beta}_{-m-\frac{k}{N}}\right)|T_k,\epsilon\rangle &= 0 & \left(\bar{\beta}_{m-\frac{k}{N}} - \tilde{\bar{\beta}}_{-m+\frac{k}{N}}\right)|T_k,\epsilon\rangle &= 0\\
\left(\eta_{r-\frac{k}{N}} - i\epsilon\tilde{\eta}_{-r+\frac{k}{N}}\right)|T_k,\epsilon\rangle &= 0 & \left(\bar{\eta}_{r+\frac{k}{N}} - i\epsilon\tilde{\bar{\eta}}_{-r-\frac{k}{N}}\right)|T_k,\epsilon\rangle &= 0\\
\left(\xi_{r+\frac{k}{N}} - i\epsilon\tilde{\xi}_{-r-\frac{k}{N}}\right)|T_k,\epsilon\rangle &= 0 & \left(\bar{\xi}_{r-\frac{k}{N}} - i\epsilon\tilde{\bar{\xi}}_{-r+\frac{k}{N}}\right)|T_k,\epsilon\rangle &= 0 \quad \text{(A.7)}
\end{aligned}$$

where $\vec{p}$ means the momentum of a D0-brane in the direction of $x^1 \sim x^5$. The solution to these constraints is

$$|T_k,\epsilon,\vec{p}\rangle_{\rm NSNS} = \exp\left[\sum_{n=1}^{\infty}\left(\frac{1}{n}\sum_{a=2}^{5}\alpha^a_{-n}\tilde{\alpha}^a_{-n}\right) + i\epsilon\sum_{r>0}\left(\sum_{a=2}^{5}\psi^a_{-r}\tilde{\psi}^a_{-r}\right)\right]\times \\ \times\exp\left[\sum_{n=0}^{\infty}\left(\frac{1}{n+\frac{k}{N}}\alpha_{-n-\frac{k}{N}}\tilde{\bar{\alpha}}_{-n-\frac{k}{N}}\right) + \sum_{n=1}^{\infty}\left(\frac{1}{n-\frac{k}{N}}\tilde{\alpha}_{-n+\frac{k}{N}}\bar{\alpha}_{-n+\frac{k}{N}}\right) + \right.\\ + \sum_{n=0}^{\infty}\left(\frac{1}{n-\frac{k}{N}}\beta_{-n+\frac{k}{N}}\tilde{\bar{\beta}}_{-n+\frac{k}{N}}\right) + \sum_{n=1}^{\infty}\left(\frac{1}{n+\frac{k}{N}}\tilde{\beta}_{-n-\frac{k}{N}}\bar{\beta}_{-n-\frac{k}{N}}\right) + \\ + i\epsilon\left(\sum_{r>0}\eta_{-r-\frac{k}{N}}\tilde{\bar{\eta}}_{-r-\frac{k}{N}} + \sum_{r>0}\tilde{\eta}_{-r+\frac{k}{N}}\bar{\eta}_{-r+\frac{k}{N}}\right) + \quad \text{(A.8)}\\ \left. + i\epsilon\left(\sum_{r>0}\xi_{-r+\frac{k}{N}}\tilde{\bar{\xi}}_{-r+\frac{k}{N}} + \sum_{r>0}\tilde{\xi}_{-r-\frac{k}{N}}\bar{\xi}_{-r-\frac{k}{N}}\right)\right]|T_i,\epsilon,\vec{p}\rangle^{(0)}_{\rm NSNS}$$

$$
\begin{aligned}
|T_k,\epsilon,\vec{p}\rangle_{RR} = \exp\Bigg[&\sum_{n=1}^{\infty}\left(\frac{1}{n}\sum_{a=2}^{5}\alpha^a_{-n}\tilde{\alpha}^a_{-n}\right)+i\epsilon\sum_{r=1}^{\infty}\left(\sum_{a=2}^{5}\psi^a_{-r}\tilde{\psi}^a_{-r}\right)\Bigg]\times\\
\times\exp\Bigg[&\sum_{n=0}^{\infty}\left(\frac{1}{n+\frac{k}{N}}\alpha_{-n-\frac{k}{N}}\tilde{\bar{\alpha}}_{-n-\frac{k}{N}}\right)+\sum_{n=1}^{\infty}\left(\frac{1}{n-\frac{k}{N}}\tilde{\alpha}_{-n+\frac{k}{N}}\bar{\alpha}_{-n+\frac{k}{N}}\right)+\\
&+\sum_{n=0}^{\infty}\left(\frac{1}{n-\frac{k}{N}}\beta_{-n+\frac{k}{N}}\tilde{\bar{\beta}}_{-n+\frac{k}{N}}\right)+\sum_{n=1}^{\infty}\left(\frac{1}{n+\frac{k}{N}}\tilde{\beta}_{-n-\frac{k}{N}}\bar{\beta}_{-n-\frac{k}{N}}\right)+\\
&+i\epsilon\left(\sum_{r=0}^{\infty}\eta_{-r-\frac{k}{N}}\tilde{\bar{\eta}}_{-r-\frac{k}{N}}+\sum_{r=1}^{\infty}\tilde{\eta}_{-r+\frac{k}{N}}\bar{\eta}_{-r+\frac{k}{N}}\right)+\\
&+i\epsilon\left(\sum_{r=1}^{\infty}\xi_{-r+\frac{k}{N}}\tilde{\bar{\xi}}_{-r+\frac{k}{N}}+\sum_{r=0}^{\infty}\tilde{\xi}_{-r-\frac{k}{N}}\bar{\xi}_{-r-\frac{k}{N}}\right)\Bigg]|T_i,\epsilon,\vec{p}\rangle^{(0)}_{\rm RR}\,.
\end{aligned}
\tag{A.9}
$$

Here $|T_i\rangle^{(0)}$ means zero mode part of the boundary state.

The GSO-projection invariant combination is

$$
|T_k\rangle_{\rm NSNS} = \frac{1}{2}\int\left(\frac{dp}{2\pi}\right)^5\left[|T_i,+,\vec{p}\rangle_{\rm NSNS}\pm|T_i,-,\vec{p}\rangle_{\rm NSNS}\right] \tag{A.10}
$$

$$
|T_k\rangle_{RR} = \frac{1}{2}\int\left(\frac{dp}{2\pi}\right)^5\left[|T_i,+,\vec{p}\rangle_{RR}\pm|T_i,-,\vec{p}\rangle_{RR}\right]. \tag{A.11}
$$

Signs in these equations are determined[7] as follows.

- $k=0$: $(-)$ for NS-sector, $(+)$ for R-sector;
- $k=N/2$: $(+)$ for NS-sector, $(+)$ for R-sector;
- Others: $(-)$ for NS-sector, $(+)$ for R-sector.

Finally, $|T_k\rangle$ is constructed from a linear combination of $|T_k\rangle_{\rm NSNS}$ and $|T_k\rangle_{RR}$ as

$$
|T_k\rangle = \frac{\mathcal{N}_k^{\rm NSNS}}{2}|T_k\rangle_{\rm NSNS}+\frac{\mathcal{N}_k^{RR}}{2}|T_k\rangle_{RR}\,, \tag{A.12}
$$

where the coefficients: $\mathcal{N}_k^{\rm NSNS}$, $\mathcal{N}_k^{RR}$ are determined from the Cardy's condition:(3.3) as we will see below.

Boundary state calculations. In order to determine the coefficients in eq. (A.12) we require Cardy's condition eq. (3.3).

[7]It is easily shown by requiring the GSO invariance that each of these sign is $+$ if there are fermionic zero mode or is $-$ if not.

Before we do that, let us define "Z" functions following ref. [23] as

$$
\begin{aligned}
Z^{\alpha}_{\beta}(\tau) &= q^{\frac{3\alpha^2-1}{24}} e^{i\pi\alpha\beta} \prod_{m=1}^{\infty} \left(1 + e^{i\pi\beta} q^{m-\frac{1-\alpha}{2}}\right)\left(1 + e^{-i\pi\beta} q^{m-\frac{1+\alpha}{2}}\right), \\
\eta(\tau) &= q^{-\frac{1}{24}} \prod_{m=1}^{\infty} (1-q^m),
\end{aligned}
\tag{A.13}
$$

where we set $q = e^{i2\pi\tau} = e^{-4\pi l}$

These functions have following modular properties

$$
Z^{\alpha}_{\beta}(\tau) = Z^{\beta}_{-\alpha}\left(-\frac{1}{\tau}\right), \qquad \eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\,\eta(\tau)\,. \tag{A.14}
$$

First we show the calculations of r.h.s. of eq. (3.3). If k is not 0, then we get

$$
\begin{aligned}
\langle +, T_k|\Delta|+, T_k\rangle_{\mathrm{NSNS}} &= V_1 \int_0^{\infty} dl \left(\frac{dk}{2\pi}\right)^5 e^{-\pi l\alpha' k^2} \frac{|Z^0_0(2il)|^2 \left|Z_0^{2\frac{k}{N}}(2il)\right|^2}{\eta(2il)^4 \left|Z_1^{1-2\frac{k}{N}}(2il)\right|^2} \\
\langle +, T_k|\Delta|-, T_k\rangle_{\mathrm{NSNS}} &= V_1 \int_0^{\infty} dl \left(\frac{dk}{2\pi}\right)^5 e^{-\pi l\alpha' k^2} \frac{|Z^0_1(2il)|^2 \left|Z_1^{-2\frac{k}{N}}(2il)\right|^2}{\eta(2il)^4 \left|Z_1^{1-2\frac{k}{N}}(2il)\right|^2} \\
\langle +, T_k|\Delta|+, T_k\rangle_{RR} &= V_1 \int_0^{\infty} dl \left(\frac{dk}{2\pi}\right)^5 e^{-\pi l\alpha' k^2} \frac{|Z^1_0(2il)|^2 \left|Z_0^{1-2\frac{k}{N}}(2il)\right|^2}{\eta(2il)^4 \left|Z_1^{1-2\frac{k}{N}}(2il)\right|^2} \\
\langle +, T_k|\Delta|-, T_k\rangle_{RR} &= 0 \qquad \text{(This is because of fermionic zero modes)},
\end{aligned}
\tag{A.15}
$$

where we defined V_1 as the volume of the Neumann direction.

Doing the Gauss integrations and modular transformations, we get

$$
\begin{aligned}
\langle T_k|\Delta|T_k\rangle &= \frac{\pi\alpha' V_1}{8\cdot(2\pi)^5} \int_0^{\infty} dl\,(l\alpha')^{-\frac{5}{2}} \frac{1}{\eta(2il)^4 \left|Z_1^{1-2\frac{k}{N}}(2il)\right|^2} \times \\
&\quad \times \left\{ |\mathcal{N}_k^{\mathrm{NSNS}}|^2 |Z^0_0(2il)|^2 \left|Z_0^{2\frac{k}{N}}(2il)\right|^2 - |\mathcal{N}_k^{\mathrm{NSNS}}|^2 |Z^0_1(2il)|^2 \left|Z_1^{-2\frac{k}{N}}(2il)\right|^2 - \right. \\
&\quad \left. - |\mathcal{N}_k^{RR}|^2 |Z^1_0(2il)|^2 \left|Z_0^{1-2\frac{k}{N}}(2il)\right|^2 \right\} \\
&= \frac{\pi(\alpha')^{-\frac{3}{2}} V_1}{2^{\frac{3}{2}}(2\pi)^5} \int_0^{\infty} dt\, t^{-\frac{3}{2}} \frac{1}{\eta(it)^4 \left|Z^1_{2\frac{k}{N}-1}(it)\right|^2} \times \\
&\quad \times \left\{ |\mathcal{N}_k^{\mathrm{NSNS}}|^2 |Z^0_0(it)|^2 \left|Z^0_{2\frac{k}{N}}(it)\right|^2 - |\mathcal{N}_k^{\mathrm{NSNS}}|^2 |Z^1_0(it)|^2 \left|Z^1_{2\frac{k}{N}}(it)\right|^2 - \right. \\
&\quad \left. - |\mathcal{N}_k^{RR}|^2 |Z^0_1(it)|^2 \left|Z^0_{2\frac{k}{N}-1}(it)\right|^2 \right\},
\end{aligned}
\tag{A.16}
$$

where we defined $t = 1/2l$.

In the same way if $k=0$ (untwisted sector) then we get

$$\begin{aligned}\langle T_0|\Delta|T_0\rangle &= \frac{\pi\alpha' V_1}{8}\int_0^\infty dl \left(\frac{dk}{2\pi}\right)^9 e^{-\pi l\alpha' k^2}\frac{1}{\eta(2il)^8}\times \\ &\quad\times\left\{|\mathcal{N}_k^{\mathrm{NSNS}}|^2|Z_0^0(2il)|^4 - |\mathcal{N}_k^{\mathrm{NSNS}}|^2|Z_1^0(2il)|^4 - |\mathcal{N}_k^{RR}|^2|Z_0^1(2il)|^4\right\} \\ &= 2^{-\frac{17}{2}}\pi^{-8}(\alpha')^{-\frac{7}{2}}V_1\int_0^\infty dt\; t^{-\frac{3}{2}}\frac{1}{\eta(it)^8}\times \\ &\quad\times\left\{|\mathcal{N}_k^{\mathrm{NSNS}}|^2|Z_0^0(it)|^4 - |\mathcal{N}_k^{\mathrm{NSNS}}|^2|Z_0^1(it)|^4 - |\mathcal{N}_k^{RR}|^2|Z_1^0(it)|^4\right\}.\end{aligned} \tag{A.17}$$

Open string calculations. Next we show the calculations of l.h.s. of eq. (3.3). If k is not 0, then we get

$$\begin{aligned}&2\int_0^\infty \frac{dt}{2t}\,\mathrm{Tr}_{\mathrm{NS-R}}\left[\frac{1+(-1)^F}{2}g^k e^{-2\pi t H_o}\right] = \\ &\quad = 2^{-\frac{5}{2}}(\pi)^{-1}(\alpha')^{-\frac{1}{2}}V_1\int_0^\infty dt\, t^{-\frac{3}{2}}\frac{C}{\eta(it)^4|Z^1_{2\frac{k}{N}-1}(it)|^2}\times \\ &\quad\times\left\{|Z_0^0(it)|^2|Z^0_{2\frac{k}{N}}(it)|^2 - |Z_1^0(it)|^2|Z^0_{2\frac{k}{N}-1}(it)| - |Z_0^1(it)|^2|Z^1_{2\frac{k}{N}}(it)|\right\},\end{aligned} \tag{A.18}$$

where C is 4 for $k=N/2$ and is 1 for others.

If k is 0, then we get

$$\begin{aligned}&2\int_0^\infty \frac{dt}{2t}\,\mathrm{Tr}_{\mathrm{NS-R}}\left[\frac{1+(-1)^F}{2}e^{-2\pi t H_o}\right] = \\ &\quad = 2^{-\frac{5}{2}}(\pi)^{-1}(\alpha')^{-\frac{1}{2}}V_1\int_0^\infty dt\, t^{-\frac{3}{2}}\cdot\frac{Z_0^0(it)^4 - Z_0^1(it)^4 - Z_1^0(it)^4}{\eta(it)^8}.\end{aligned} \tag{A.19}$$

Determination of the normalization. Now we can determine the coefficients in eq. (A.12) by using Cardy's condition eq. (3.3).

Then the results are as follows:

1. $k=0$ case: $\mathcal{N}_0^{\mathrm{NSNS}} = \mathcal{N}_0^{RR} = 2^3\pi^{7/2}\alpha'^{3/2}$,

2. $k=N/2$ case: $\mathcal{N}_0^{\mathrm{NSNS}} = \mathcal{N}_0^{RR} = 2^4\pi^{3/2}\alpha'^{1/2}$,

3. $k\neq 0, N/2$ case: $2\mathcal{N}_0^{\mathrm{NSNS}} = \mathcal{N}_0^{RR} = 2^3\pi^{3/2}\alpha'^{1/2}$.

References

[1] J. Dai, R.G. Leigh and J. Polchinski, *New connections between string theories*, *Mod. Phys. Lett.* **A 4** (1989) 2073;
J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [`hep-th/9510017`].

[2] M.R. Douglas and G. Moore, *D-branes, quivers and ale instantons*, `hep-th/9603167`.

[3] C.V. Johnson and R.C. Myers, *Aspects of type-IIB theory on ALE spaces*, *Phys. Rev.* **D 55** (1997) 6382 [hep-th/9610140].

[4] P.S. Aspinwall, *Resolution of orbifold singularities in string theory*, hep-th/9403123; *Enhanced gauge symmetries and K3 surfaces*, *Phys. Lett.* **B 357** (1995) 329 [hep-th/9507012].

[5] J.D. Blum and K. Intriligator, *Consistency conditions for branes at orbifold singularities*, *Nucl. Phys.* **B 506** (1997) 223 [hep-th/9705030].

[6] D.-E. Diaconescu, M.R. Douglas and J. Gomis, *Fractional branes and wrapped branes*, *J. High Energy Phys.* **02** (1998) 013 [hep-th/9712230].

[7] M. Billò, B. Craps and F. Roose, *On D-branes in type-0 string theory*, *Phys. Lett.* **B 457** (1999) 61 [hep-th/9902196].

[8] D.-E. Diaconescu and J. Gomis, *Fractional branes and boundary states in orbifold theories*, hep-th/9906242.

[9] A. Recknagel and V. Schomerus, *D-branes in Gepner models*, *Nucl. Phys.* **B 531** (1998) 185 [hep-th/9712186].

[10] M. Gutperle and Y. Satoh, *D-branes in Gepner models and supersymmetry*, *Nucl. Phys.* **B 543** (1999) 73 [hep-th/9808080]; *D0-branes in Gepner models and $N = 2$ black holes*, *Nucl. Phys.* **B 555** (1999) 477 [hep-th/9902120].

[11] I. Brunner, M.R. Douglas, A. Lawrence and C. Römelsberger, *D-branes on the quintic*, hep-th/9906200.

[12] M.R. Douglas and B. Fiol, *D-branes and discrete torsion, 2*, hep-th/9903031.

[13] M.B. Green, *Pointlike states for type-IIb superstrings*, *Phys. Lett.* **B 329** (1994) 435 [hep-th/9403040];
M.B. Green and M. Gutperle, *Light-cone supersymmetry and D-branes*, *Nucl. Phys.* **B 476** (1996) 484 [hep-th/9604091].

[14] J.L. Cardy, *Boundary conditions, fusion rules and the Verlinde formula*, *Nucl. Phys.* **B 324** (1989) 581.

[15] P.D. Vecchia M. Frau, A. Lerda, I. Pesando, R. Russo and S. Sciuto, *Classical p-branes from boundary state*, *Nucl. Phys.* **B 507** (1997) 259 [hep-th/9707068].

[16] C.P. Bachas, M.R. Douglas and M.B. Green, *Anomalous creation of branes*, *J. High Energy Phys.* **07** (1997) 002 [hep-th/9705074].

[17] O. Bergman, M.R. Gaberdiel and G. Lifschytz, *Branes, orientifolds and the creation of elementary strings*, *Nucl. Phys.* **B 509** (1998) 194 [hep-th/9705130];
U. Danielsson, G. Ferretti and I.R. Klebanov, *Creation of fundamental strings by crossing D-branes*, *Phys. Rev. Lett.* **79** (1997) 1984 [hep-th/9705084].

JHEP02(2000)040

[18] S. Kachru and E. Silverstein, *4d conformal theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183];
A. Lawrence, N. Nekrasov and C. Vafa, *On conformal field theories in four dimensions*, *Nucl. Phys.* **B 533** (1998) 199 [hep-th/9803015].

[19] I.R. Klebanov and N.A. Nekrasov, *Gravity duals of fractional branes and logarithmic RG flow*, hep-th/9911096.

[20] E. Witten, *Solutions of four-dimensional field theories via M-theory*, *Nucl. Phys.* **B 500** (1997) 3 [hep-th/9703166].

[21] C.V. Johnson, A.W. Peet and J. Polchinski, *Gauge theory and the excision of repulson singularities*, hep-th/9911161.

[22] J. McKay, *Graphs, singularities and finite groups*, *Proc. Symp. Pure Math.* **37** (1980) 183.

[23] J. Polchinsky, *String theory, I II*, Cambridge University Press, 1998.

[24] O. Bergman and M.R. Gaberdiel, *Stable non-BPS D-particles*, *Phys. Lett.* **B 441** (1998) 133 [hep-th/9806155];
A. Sen, *BPS D-branes on non-supersymmetric cycles*, *J. High Energy Phys.* **12** (1998) 021 [hep-th/9812031].

[25] O. Bergman and M.R. Gaberdiel, *A non-supersymmetric open-string theory and s-duality*, *Nucl. Phys.* **B 499** (1997) 183 [hep-th/9701137].